ST. JOSEPH OF NAZARETH HIGH SCHOOL

MATHEMATICS DEPARTMENT- MARKING SCHEME

SUBJECT: SUB-MATHS - INTERNAL MOCK PAPER: S475/2

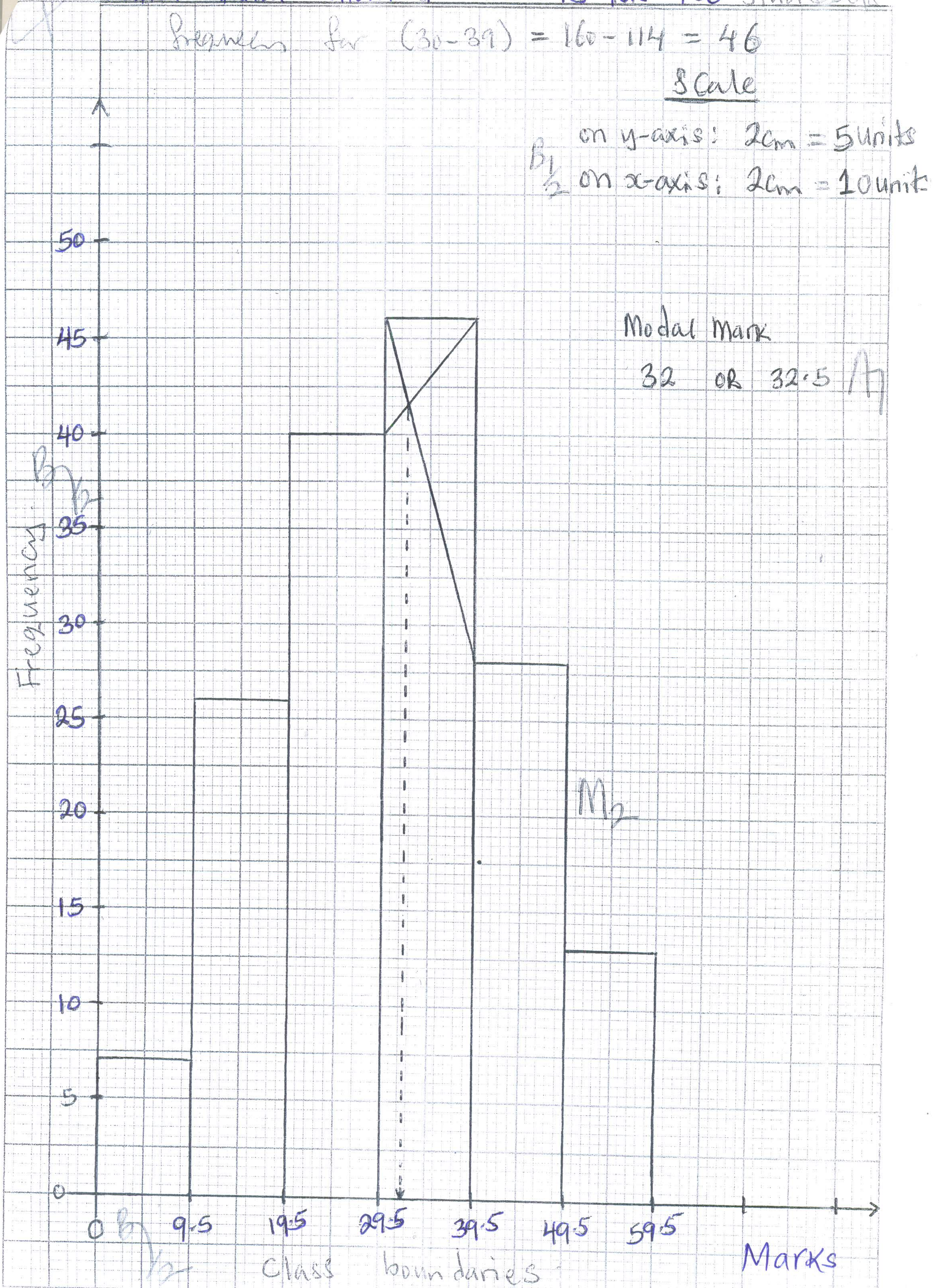
CLASS: S.6 TERM: II EXAM: INT-MOCK YEAR: 2014

NO.	SOLUTION	MKS	COMMENT
1.	(i) $(3\underset{\sim}{a}+\underset{\sim}{b})\cdot\underset{\sim}{b}$ $(3\underset{\sim}{a}+\underset{\sim}{b}) = 3\begin{pmatrix}3\\-4\end{pmatrix}+\begin{pmatrix}-5\\12\end{pmatrix}=\begin{pmatrix}9\\-12\end{pmatrix}+\begin{pmatrix}-5\\12\end{pmatrix}$ $(3\underset{\sim}{a}+\underset{\sim}{b}) = \begin{pmatrix}4\\0\end{pmatrix}$ $(3\underset{\sim}{a}+\underset{\sim}{b})\cdot\underset{\sim}{b} = (4\underset{\sim}{i}+0\underset{\sim}{j})\cdot(-5\underset{\sim}{i}+12\underset{\sim}{j})$ $= -20+0$ $\therefore (3\underset{\sim}{a}+\underset{\sim}{b})\cdot\underset{\sim}{b} = -20$		
	(ii) $\underset{\sim}{a}\cdot\underset{\sim}{b} = \lvert\underset{\sim}{a}\rvert\cdot\lvert\underset{\sim}{b}\rvert\cos\theta$ $\begin{pmatrix}3\\-4\end{pmatrix}\cdot\begin{pmatrix}-5\\12\end{pmatrix} = \left\lvert\begin{pmatrix}3\\-4\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}-5\\12\end{pmatrix}\right\rvert\cos\theta$ $-15-48 = \left(\sqrt{3^2+(-4)^2}\right)\left(\sqrt{(-5)^2+12^2}\right)\cos\theta$ $-63 = 5\times 13\cos\theta$ $\cos\theta = \frac{-63}{65}$ $\theta = 165.75°$ $\therefore$ Angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $165.75°$		

NO. 2

A HISTOGRAM SHOWING RESULTS FOR 160 Students

Frequency for (30-39) = 160 - 114 = 46



NO.	SOLUTION	MKS	COMMENT
3.	$U_{21} = a + 20d = 252$ $a + 20d = 252$ $52 + 20d = 252$ $20d = 200$ $d = 10$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{16} = \frac{16}{2}[2 \times 52 + 15 \times 10]$ $\therefore S_{16} = 2032$		
4.	(table below)		

P_0	P_1	W	P_0W	P_1W
400	500	7	2800	3500
900	1100	2	1800	2200
600	700	3	1800	2100
600	800	6	3600	4800
			$\Sigma P_0W = 10{,}000$	$\Sigma P_1W = 12{,}600$

$$W.P.I = \frac{\Sigma P_1W}{\Sigma P_0W} \times 100$$

$$= \frac{12600}{10{,}000} \times 100\%$$

$$= 126$$

$(126 - 100) = 26\%$, meaning that the prices increased by 26% in 1996 compared to 1994.

NO.	SOLUTION	MKS	COMMENT
5.	(a) let $y = (x+5)^2(x-1)$ $y = (x^2+10x+25)(x-1)$ $y = x^3+9x^2+15x-25$ $\frac{dy}{dx} = 3x^2+18x+15.$ $\therefore \frac{d}{dx}\left[(x+5)^2(x-1)\right] = 3x^2+18x+15$ (b) let $y = \frac{1}{x^2} + x^2 = x^{-2}+x^2$ $\frac{dy}{dx} = -2x^{-3}+2x = \frac{-2}{x^3}+2x$ $\therefore \frac{d}{dx}\left(\frac{1}{x^2}+x^2\right) = 2x - \frac{2}{x^3}$		
6.	$C = AB = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 5 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 11 \\ -2 & 10 \end{pmatrix}$ $\det(C) = 2\times10 - {-22} = 42$ $Adj(C) = \begin{pmatrix} 10 & -11 \\ 2 & 2 \end{pmatrix}$ $C^{-1} = \frac{1}{42}\begin{pmatrix} 10 & -11 \\ 2 & 2 \end{pmatrix}$ $\therefore C^{-1} = \begin{pmatrix} 5/21 & -11/42 \\ 1/21 & 1/21 \end{pmatrix}$		

NO.	SOLUTION	MKS	COMMENT
7.	Saloon cars $= 6$ Vans $= 7$ Atleast 3 saloon cars $= ({}^6C_3 \times {}^7C_2) + ({}^6C_4 \times {}^7C_1) + ({}^6C_5 \times {}^7C_0)$ $= (20 \times 21) + (15 \times 7) + (6 \times 1)$ $= 420 + 105 + 6$ $= 531$ ways		
8.	$u = ?$, $V = 40m/s$, $a = 5m/s^2$ $t = 6s$ (i) $V = u + at$ $40 = u + 5 \times 6$ $40 = u + 30$ $\therefore u = 40 - 30 = 10m/s$		
	(ii) $S = ut + \frac{1}{2}at^2$ $S = 10 \times 6 + \frac{5}{2} \times 6^2$ $S = (60 + 90) = 150m$		
	Alternatively: $V^2 = u^2 + 2as$ $40^2 = 10^2 + 2 \times 5 \times S$ $1600 = 100 + 10s$ $\frac{1500}{10} = \frac{10s}{10}$ $\therefore S = 150m$		Using the 3rd equation of motion.

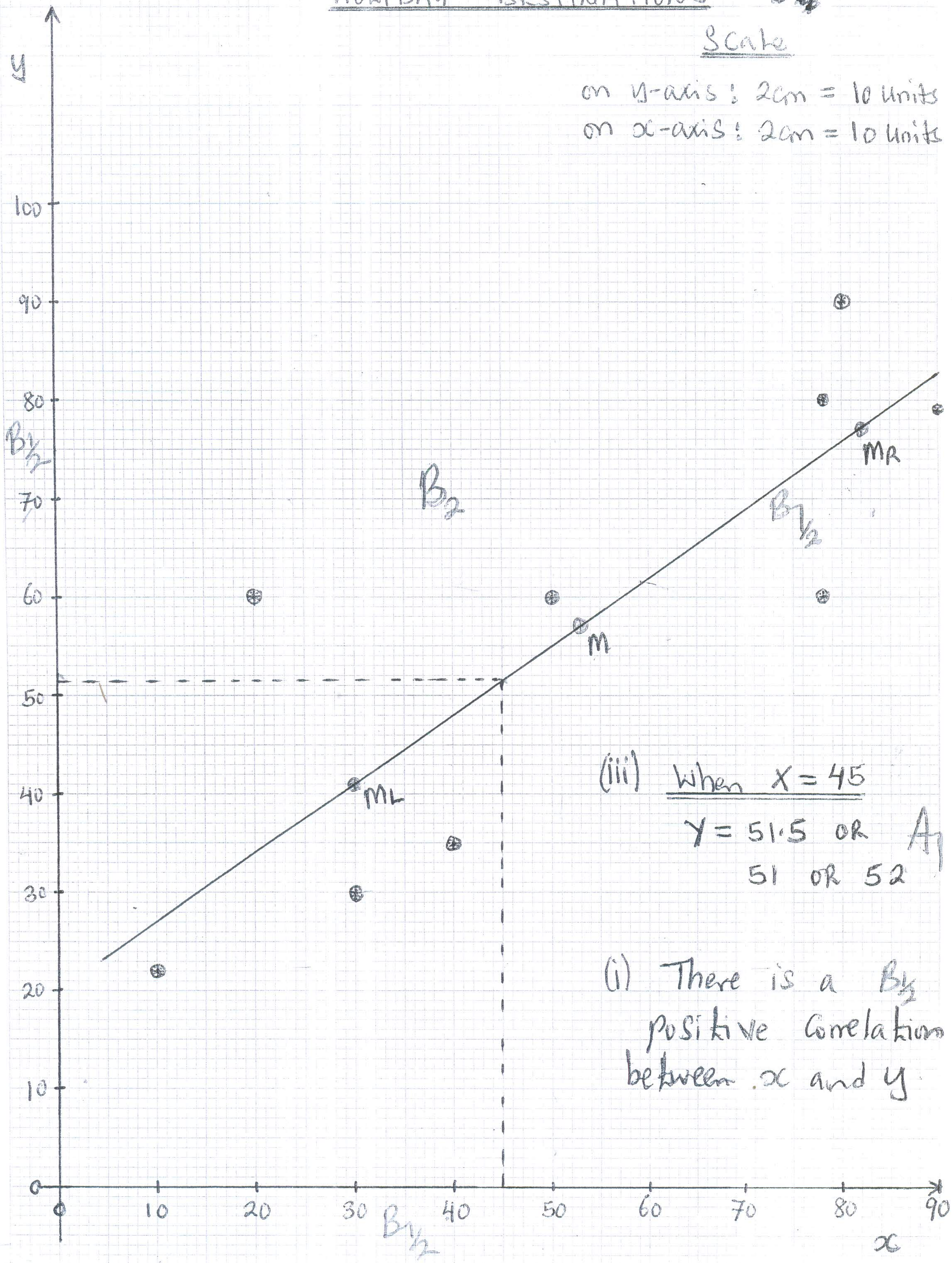
SECTION B

NO.	SOLUTION	MKS	COMMENT
9.	(a)(i) See the graph overleaf. Comment: There is a positive correlation between x and y. (ii) Line of best fit $M(\bar{x},\bar{y})$ $\bar{x} = \frac{90+80+78+78+50+40+30+20+10}{9}$ $\bar{x} = \frac{476}{9} = 52.9 \approx 53$ $\bar{y} = \frac{79+90+80+60+60+35+30+60+22}{9}$ $\bar{y} = \frac{516}{9} = 57.3 \simeq 57$ $\therefore M(\bar{x},\bar{y}) = (53, 57)$		
	$M_L(\bar{x},\bar{y})$ $\bar{x} = \frac{10+30+40+50+20}{5} = 30$ $\bar{y} = \frac{22+60+30+35+60}{5} = 41.4 \approx 41$ $M_L(\bar{x},\bar{y}) = (30, 41)$		
	$M_R(\bar{x},\bar{y})$ $\bar{x} = \frac{78+78+80+90}{4} = 81.5 \simeq 82$ $\bar{y} = \frac{60+80+90+79}{4} = 77.3 \simeq 77$ $M_R(\bar{x},\bar{y}) = (82, 77)$		

A SCATTER DIAGRAM SHOWING NINE MOST POPULAR HOLIDAY DESTINATIONS

Scale

on y-axis: 2cm = 10 units
on x-axis: 2cm = 10 units



(iii) When $X = 45$

$Y = 51.5$ OR 51 OR 52

(i) There is a positive correlation between x and y

NO.	SOLUTION	MKS	COMMENT
9.	(b) Using $\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$		

x	y	R_x	R_y	$d = R_x - R_y$	d^2
90	79	1	3	-2	4
80	90	2	1	1	1
78	80	3.5	2	1.5	2.25
78	60	3.5	5	-1.5	2.25
50	60	5	5	0	0
40	35	6	7	-1	1
30	30	7	8	-1	1
20	60	8	5	3	9
10	22	9	9	0	0
					$\sum d^2 = 20.5$

$$\rho = 1 - \frac{6 \times 20.5}{9(9^2-1)}$$

$$\rho = 1 - \frac{123}{720}$$

$$= 1 - 0.17083$$

$$= 0.82917$$

$$\therefore \rho = 0.829 \quad (3 \text{ d.p})$$

Comment:

There is a very high positive correlation.

NO.	SOLUTION	MKS	COMMENT
10 (a) (i)	$ABC = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} -9 & 15 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $ABC = \begin{pmatrix} -3 & 66 & 39 \\ 4 & 2 & -2 \end{pmatrix}$		
(ii)	$(A+B)C = \left[\begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}\right]\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 10 & 23 & 5 \\ 4 & 11 & 3 \end{pmatrix}$		
10(b)	$\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 8 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 6-4 & 8-8 \\ -3+3 & -4+6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16-12 \\ -8+9 \end{pmatrix}$ $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $2x = 4$ $x = 2$ $2y = 1$ $y = \frac{1}{2}$ $\therefore\ x = 2,\quad y = \frac{1}{2}$		

No. 11

NO.	SOLUTION	MKS	COMMENT
11.	(a)(i) $\int_0^1 kx\,dx + \int_1^2 \left(\frac{k}{2}\right) x\,dx = 1$ $k\left[\frac{x^2}{2}\right]_0^1 + \frac{k}{2}\left[\frac{x^2}{2}\right]_1^2 = 1$ $\frac{k}{2} + \frac{k}{2}\left[\frac{3}{2}\right] = 1$ $\frac{2k+3k}{4} = 1$ $\frac{5k}{4} = 1$ $k = \frac{4}{5}$		
	(a)(ii) $E(x) = \int x f(x)\,dx$ $E(x) = k\int_0^1 x^2\,dx + \frac{k}{2}\int_1^2 x^2\,dx$ $= k\left[\frac{x^3}{3}\right]_0^1 + \frac{k}{2}\left[\frac{x^3}{3}\right]_1^2$ $= \frac{4}{5}\left[\frac{1}{3}\right] + \frac{2}{5}\left[\frac{7}{3}\right]$ $= \frac{4}{15} + \frac{14}{15} = \frac{18}{15} = \frac{6}{5}$ $\therefore E(x) = 1.2$		
	a(iii) $\int_{-\infty}^{m} f(x)\,dx = 0.5$ $\int_0^1 f(x)\,dx = \int_0^1 kx\,dx = \int_0^1 \frac{4}{5}x\,dx = \frac{2}{5} = 0.4$ Since $0.4 < 0.5$ $\therefore \int_0^1 kx\,dx + \int_1^m \frac{k}{2}x\,dx = 0.5$		

NO.	SOLUTION	MKS	COMMENT
	$\frac{4}{5}\int_0^1 x\,dx + \frac{2}{5}\int_1^m x\,dx = \frac{1}{2}$		
	$\frac{2}{5} + \frac{2}{5}\left[\frac{x^2}{2}\right]_1^m = \frac{1}{2}$		
	$\frac{2}{5} + \frac{2}{5}\left[\frac{m^2}{2} - \frac{1}{2}\right] = \frac{1}{2}$		
	Multiplying by 10.		
	$4 + 2m^2 - 2 = 5$		
	$2m^2 = 3$		
	$m^2 = \frac{3}{2}$		
	$m = \sqrt{1.5}$		
	$m = 1.22247$		
	$m \simeq 1.225$ (3dp)		
	$\therefore$ The median is 1.225 since its > 0.5.		
11.(b)	$P(\frac{1}{2} \leq X \leq 1\frac{1}{2}) = P(0.5 \leq X \leq 1.5)$		
	$= \int_{0.5}^1 kx\,dx + \frac{k}{2}\int_1^{1.5} x\,dx$		
	$= \frac{4}{5}\left[\frac{x^2}{2}\right]_{1/2}^1 + \frac{2}{5}\left[\frac{x^2}{2}\right]_1^{1.5}$		
	$= \frac{4}{5}\left[\frac{1}{2} - \frac{0.25}{2}\right] + \frac{2}{5}\left[\frac{2.25}{2} - \frac{1}{2}\right]$		
	$= \frac{4}{5}(0.375) + \frac{2}{5}(0.625)$		
	$= \frac{4}{5}\left(\frac{3}{8}\right) + \frac{2}{5}\left(\frac{5}{8}\right)$		
	$= \frac{3}{10} + \frac{1}{4}$		
	$= \frac{11}{20}$ or 0.55		

NO.	SOLUTION	MKS	COMMENT
12.	(a) $\int_0^6 (2x^3+6x)\,dx = \left[\frac{2x^4}{4}+\frac{6x^2}{2}\right]_0^6$ $= \left[\frac{1}{2}x^4+3x^2\right]_0^6$ $= 756 - 0$ $= 756$		
	b). $y = 5+4x-x^2$ $\frac{dy}{dx} = 4-2x$ For turning point, $\frac{dy}{dx}=0$ $4-2x=0$ $x=2$ For $x=2$, $y=5+4(2)-(2)^2=9$ $(x,y)=(2,9)$. Turning point $(2,9)$ Nature: $\frac{dy}{dx}=4-2x$ $\frac{d^2y}{dx^2}=-2$ Since $\frac{d^2y}{dx^2}<0$; the point is a maximum. Intercepts: For y-intercept, $x=0$, $y=5$ For x-intercept, $y=0$		

NO.	SOLUTION	MKS	COMMENT

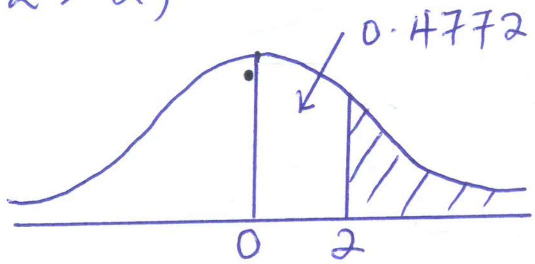
13.(a)(i) $\mu = 50$; $\sigma = 10$

Using $z = \dfrac{x-\mu}{\sigma}$

$P(x>70) = P(z>z_1)$

$z_1 = \dfrac{70-50}{10} = \dfrac{20}{10} = 2$

$P(z>2)$



$P(z>2) = 0.5 - 0.4772 = 0.0228$

$P(x>70) = 0.0228$

Percentage $= 0.0228 \times 100\%$

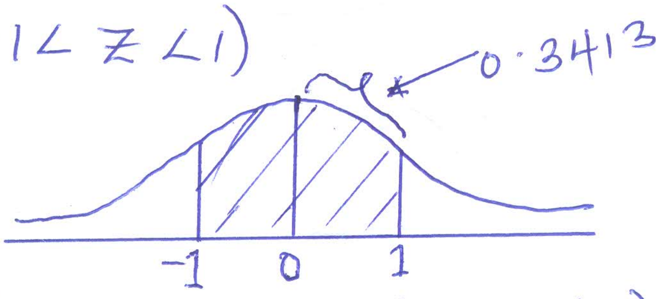
$= 2.28\%$

(ii) $P(40 < x < 60) = P(z_1 < z < z_2)$

$z_1 = \dfrac{40-50}{10} = \dfrac{-10}{10} = -1$

$z_2 = \dfrac{60-50}{10} = \dfrac{10}{10} = 1$

$P(-1 < z < 1)$



$= P(-1 < z < 0) + P(0 < z < 1)$

$= P(0 < z < 1) + P(0 < z < 1)$

$= 0.3413 + 0.3413$

$= 0.6826$

$P(40 < x < 60) = 0.6826$

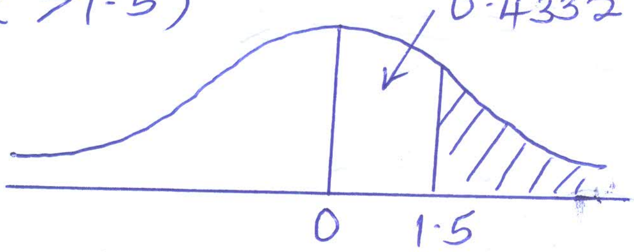
NO.	SOLUTION	MKS	COMMENT

13. Percentage $= 0.6826 \times 100\% = 68.26\%$

(b)(i) $P(X > 65) = P(Z > Z_1)$

$$Z_1 = \frac{65-50}{10} = \frac{15}{10} = 1.5$$

$P(Z > 1.5)$



$P(Z > 1.5) = 0.5 - 0.4332$

$= 0.0668$

No of candidates > 65

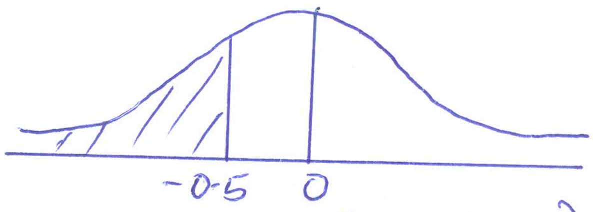
$= 0.0668 \times 10000$

$= 668$

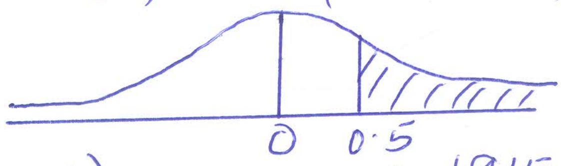
(b)(ii) $P(X < 45) = P(Z < Z_1)$

$$Z_1 = \frac{45-50}{10} = \frac{-5}{10} = -0.5$$

$P(Z < -0.5)$



$P(Z < -0.5) = P(Z > 0.5)$



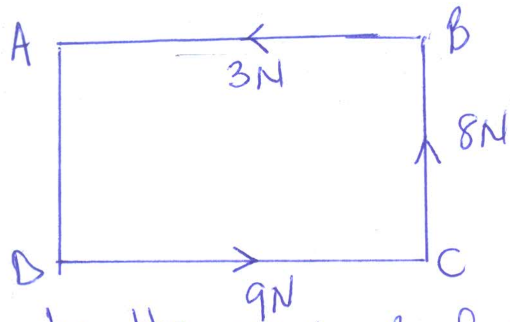
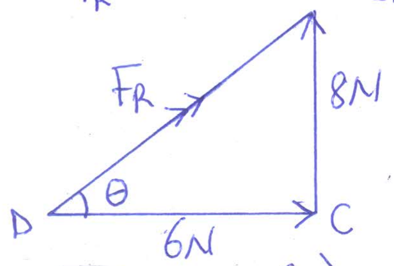
$P(Z > 0.5) = 0.5 - 0.1915$

$= 0.3085$

No. of candidates $= 0.3085 \times 10000$

$= 3085$

NO.	SOLUTION	MKS	COMMENT
14-	(a) <u>Vertical forces</u> $1 \sin 80° = 0.985N$ (↑) $4 \sin 30° = 2N$ (↑) $p \cos \theta$ (↓) $2N$ (↓) <u>At equilibrium point</u>: upward forces = downwards $0.985 + 2 = p\cos\theta + 2$ $p\cos\theta = 0.985$ $\cos\theta = \frac{0.985}{p}$ ---------- (i) <u>Horizontal forces</u> $1 \cos 80° = 0.174N$ (←) $4 \cos 30° = 3.464N$ (→) $p \sin\theta$ (←) $3N$ (→) <u>At equilibrium</u> $p\sin\theta + 0.174 = 3.464 + 3$ $p\sin\theta = 6.29$ $\sin\theta = \frac{6.29}{p}$ ---------- (ii) Using eqn's (i) and (ii) $\tan\theta = \frac{6.29}{p} \div \frac{0.985}{p}$ $\tan\theta = \frac{6.29}{0.985}$ $\tan\theta = 6.3858$ $\theta = \tan^{-1} 6.3858$ $\theta = 81.1°$		

NO.	SOLUTION	MKS	COMMENT
14(a) (a)	$p \sin\theta = 6.29$ $p = \frac{6.29}{\sin 81.1^\circ}$ $p = 6.4N$		
14(b)	 Let F_R be the resultant force;  $(9 - 3 = 6N)$ $F_R = \sqrt{6^2 + 8^2}$ $F_R = \sqrt{100}$ $F_R = 10N$.		
	$\tan\theta = 8/6$ $\tan\theta = 1.3333$ $\theta = \tan^{-1} 1.3333$ $\theta = 53.1^\circ$		